# 3 Conclusion

The purpose of these self-contained notes is to highlight some new aspects of interference using pre- and post-selection and weak measurements. We have emphasized the importance of analyzing quantum mechanical interference through the use of states which are more 'non-classical,' namely with states comprised of multiple lumps. We argued that the most appropriate picture for these situations is given by modular variables. Exciting new features of quantum mechanics were obtained by using this approach. For example, applying the Heisenberg picture to multiple-lump states lead us to a **physical** explanation for the different behaviors of a *single* particle when the distant slit is open or closed: this helps us to shift from a picture involving a quantum wave that passes through all slits, to one with more 'localized' particles which interact *non-locally* with the other slit(s). This contrasts with classical mechanics where the equations of motion are local.

Furthermore, we showed that although particles localized around the right slit can exchange modular momentum (non-locally) with the "barrier" at the left slit, the uncertainty in quantum mechanics appears to be just right to protect causality. In other words, exactly under the conditions when the non-local exchange of modular variables *could potentially* violate causality, then the variable exchanged nonlocally becomes completely uncertain and therefore unobservable, thereby protecting causality. Nevertheless, we showed that the nonlocal equations can have a measureable effect without violating causality. We showed this with the following calculation: if the left slit is later closed, then the probability that the earlier weak measurement shifts any particles from the right slit to the left slit is $O(\frac{1}{N})$. Therefore, in the limit of large $N$, the weak measurement does not shift even a single particle from the right slit to the left slit. This can be confirmed by placing a photographic plate at the left slit. On the other hand, if the left slit is later opened (i.e. after the weak measurement), then we calculated that a small number of particles (independent of $N$), are shifted from the right slit to the left slit. However, all $N$ particles contribute to the dramatically different weak measurement results. This is explained by the nonlocal equations of motion satisfied by the modular variables[5].

Finally, we note that while the Heisenberg picture and the Schrödinger pictures are equivalent formulations of quantum mechanics, nevertheless, the results discussed here support a new approach which has led to new insights, new intuitions, new experiments, and even the possibility of new devices that were missed from the old perspective. These types of developments are signatures of a successful re-formulation.

[5] These observables, $p \, mod \, \frac{h}{D}$, do not have a classical limit: e.g. when $D$ is kept fixed and $\hbar \to 0$, then these operators oscillate infinitely fast. Therefore, although these operators continue to obey non-local equations of motion in the classical limit, the non-locality loses its observable meaning in the classical limit.