Grothendieck's *Promenade*, or the Eulogy of Solitude.

An Introduction to Grothendieck's Spirit by His Own Words

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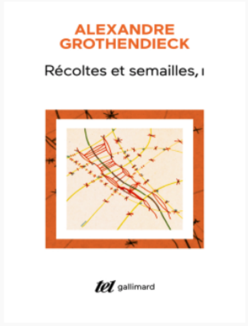

Alexandre Grothendieck, *Récoltes et semailles. Réflexions et témoignages sur un passé de mathématicien*, Gallimard, Paris, 2021 (2 vols)

- I. En guise d'avant-propos, January 30$^{\text{th}}$, 1986, vol. I, pp.10-15.
- II. Promenade à travers un Œuvre — ou l'enfant et la mère, January, 1986, vol. I, pp. 16-96.

Youthful Memories: The Magic of Learning

Let us begin with this:

> [P.1-2] *When I was a kid, I loved to go to school. <...> I don't remember we never be bored in school at that time. There was the magic of numbers, and the magic of words, signs and sounds.*
>
> *The first year of high school in France, in 1940, I was interned with my mother in the concentration camp of Rieucros, near Mende. It was wartime, and we were foreigners — "undesirables", as we were dubbed. But the camp administration kept an eye on the kids in the camp <...>. We came and left as we wished, roughly. I was the oldest, and the only one to go to the high school, four or five kilometers away, whether it was snowing or windy, with makeshift shoes that always got water.*

> *In the last years of the war, while my mother remained interned in the camp, I was in a children's home of the "Secours Suisse", for refugee children, in Chambon-sur-Lignon. Most of us were Jewish, and when we were warned (by the local police) that there would be Gestapo raids, we did run and hide in the woods for a night or two, in small groups of two or three, without really realizing that our lives were at stake.*

The passion for and the magic of learning can be stronger than a terrible environment and make even possible to disregard it and its danger. It makes us live an interior life, greatly independent of the material one.

G. tells it as if it were a quite natural thing. But what makes it the case in some of us, and not in others? Is this a purely intellectual state? Or is it rather, or also, and affective state?

Whatever it might be, is this a necessary condition for being mathematicians, or, more generally, genuine intellectuals?

Childhood Memories: The First Foundational Problem

> [P.3] *What I found most unsatisfactory in our math books was the absence of any serious definition of the notion of length (of a curve), of area (of a surface), of volume (of a solid). I promised myself to fill this gap, as soon as I had the time. I spent most of my energy on it between 1945 and 1948, while I was a student at the University of Montpellier.*
>
> *The intuition of the* ***volume*** *was unassailable. It could only be the reflection of a* ***reality****, elusive for the moment, but perfectly reliable. The question was simply that of grasping this reality.*

Is it the aim of maths that of grasping an "unassailable reality"?

- Apparently, yes, for G.

The Gift of Solitude

[P.5] *When I finally got in touch with the mathematical world in Paris <. . . >, I ended up learning there <. . . > that the work I had done in my corner with the means at hand, was (more or less) what was well known to "everybody", under the name of 'Lebesgue's measure and integral theory'. <. . . >*

Yet, looking back on those three years now, I realize that they were by no means wasted. Without even knowing it, I learned then in solitude what is the essence of being a mathematician—what no master can really teach. <. . . > These years of solitude laid the foundation for a confidence that was never shaken <. . . >. To put it another way: I have learned, in these crucial years, ***to be alone****. By this I mean: to approach by my own lights the things I want to know, rather than to rely on the ideas and consensus, expressed or tacit, that would come to me from a more or less extended group of which I would feel a member <. . . >.*

[P.63-64] *The most direct filiation that I believe to recognize at present with a mathematician of the past, is the one that links me to Évariste Galois. <...> <One of the> reason<s>, surely, that contributes to giving me this feeling of an "essential kinship" < is that> Galois remained during his lifetime, as I did a century and a half later, a "marginal" in the official mathematical world.*

[P.64-65] *The proximity I'm speaking of is that of a certain "naivety", or "innocence" <...>. It is expressed by a propensity <...> to look at things through one's own eyes, rather than through secured glasses, graciously offered by some human group, more or less large, invested with authority <...>.*

<As> we might also call it<, this is> ***the gift of solitude****.*

- Is Mathematics a social or an individual enterprise?

What G seems to suggest is that

- Routine mathematics is social, but
- Original, or innovative mathematics is essentially individual.
    - Better, it requires solitude.

More than that: G takes

- Solitude to be not a contingent circumstance, but rather a cognitive gift, an innate posture of spirit.

G is not speaking of genius here, suggests he is thinking at that. This makes him suggest that

- Mathematical genius is the ability of taking advantage of the gift of solitude.

The Mathematical Work

[A.2] *<The> mathematical work <...> <as well as> any creative work, all work of discovery <...><or at least> any work which is dubbed 'intellectual', that which is done above all "by the mind", and by writing <...> is marked by the hatching out and the blossoming of a piece of* ***understanding*** *of the things that we are probing. But, to take an example at the opposite end of the spectrum, the passion of love is also a drive to discover. It opens us to a knowledge dubbed 'carnal', which also renews itself, blossoms forth, deepens. These two drives <...> are much closer than we generally suspect, or than we are willing to admit to ourselves.*

There is no mathematics without understanding.

- But understanding is both:
  - understanding of something that is (previously) given: Platonism?
  - and a sort of internal force, a "drive <*pulsion*>" towards discovery: Intentionality, in a phenomenological sense?

History of Science/Maths

[A.2] *<Face to the> great "Myth of Science" (with capital S please!)<...> <, the> heroic,"Promethean" myth, into which writers and scientists have fallen (and continue to fall) one more than another<...>, only historians, perhaps, sometimes resist <...>.*

History of Science is the only antidote against a mythological vison of science.

More than that:

> [P.11] *Most mathematicians <...> are inclined to confine themselves to a conceptual framework, a "**Universe**" that is fixed once for all—the one, essentially, that they found "ready-made" at the time they did their studies. <...> How this <...> has been built over the generations, and how and why such and such tools have been designed and made (and not others...) <...>—these are all questions that these heirs don't dream of asking. This is the "Universe", the "given" in which one must live, full stop! Something that seems large <...>, but also **familiar**, and above all: **immutable**.*

Regardless whether this is true for most or only same mathematicians, the point here is that routine mathematics seems, then, to go together with lack of historical awareness.

- Since historical awareness makes the non-ineluctability and non-immutability of the present framework clear.

Points of View, Themes and Visions

[P.15-16] *But even more than towards <. . . > new questions, notions and statements, my particular genius pushes me towards <. . . > fruitful **points of view**, constantly leading me to introduce and develop entirely new **themes**. <. . . > <The> innumerable questions, notions, and statements <. . . ><that I have introduced> make sense for me only in the light of such a "point of view"—or to better say, they born from it spontaneously, with the force of evidence <. . . >.*

[P.16] *<. . .> the fertile points of view are, in our art, the most powerful tools of discovery, <. . .> they are the very* ***eyes*** *of <. . .><mathematicians><. . .> which at once make us* ***discover*** *and* ***recognize the unity*** *in the multiplicity of what is discovered.*

[P.19-20] *s There are <. . .> points of view which are broader than others, and which alone give rise to and encompass a multitude of partial points of view, <. . .>. Such a point of view can also be called, rightly, a "****great idea****". By its own fecundity, such an idea gives birth to a teeming progeny, of ideas which all inherit its fecundity, but most (if not all) of which are less far-reaching than the mother idea.*

[P.16-17] *But <...>, taken as such, a "point of view" remains fragmentary. It discloses* ***a single aspect*** *of a landscape or panorama, among a multiplicity of others equally valuable and "real". Only when complementary points of view on the same reality join to each another, when the "eyes" are multiplied, our sight penetrates further in the knowledge of the things.*

*And it happens, sometimes, that a beam of converging points of view on the same vast landscape <...> gives shape to a new thing; to a thing that surpasses each of the partial perspectives <...>. This new thing may be dubbed '****vision****'. The vision unites the already known points of view that embody it, and it reveals others hitherto ignored <...>.*

Mathematics discovers (it does not invent).

- And innovative discoveries are discoveries of unities, or, even better, unities of unities.
  - A point of view unites several questions, notions and statements;
  - Some points of view are larger than others: they are great ideas, generate a progeny of subordinate point of views. They display what others have called 'germinal richness', and this makes them engender a single theme;
  - Still, a point of view, or theme only discloses an aspect of the relevant piece of reality;
  - Several distinct points of view, or themes shape a vision which unites them, and the corresponding aspects within a single sight.
  - On the strength of a vision, the mathematician discerns new aspects that were beforehand ignored.

More than being unitary, mathematics advances, then, by unification; and unification does not only brings things together, but also reveals things. It works, as a lens that fine-tunes our sight without restricting, but rather enlarging the horizon.

- This is what G seems to intend by foundation.

Foundation

Here is, indeed, what he wrote later:

> [P.20; n.] *The part of my program on the schematic theme and its extensions and ramifications $<\ldots>$ represents in itself the most extensive foundational work ever accomplished in the history of mathematics and surely one of the most extensive in the history of science.*

The question of the foundation of mathematics is not, then, for G the same as the problem of providing some sort of internal (or even external) justification of it, but rather that of

- unifying it, by providing new perspectives for its future unitary development.
    - As obvious as this might appear to professional mathematicians, it is quite worthwhile to notice it for philosophers and historians of mathematics and its philosphy.
- Since this opens a quite different perspective on what should count as the history and practice of foundation of mathematics.

G's Foundational Program

[P.17; n.] <. . . > *I have the impression that in* <. . . >< *my work program*> *the "twists"* <. . . > *are only matter of detail, generally quickly spotted by my own care. They are simple "mishaps along the way", of a purely "local" nature and without serious incidence on the validity of the essential intuitions* <. . . >. *But, at the level of the ideas and the great guiding intuitions, it seems to me that my work is free of any "miss"* <. . . >. <*My*> *assurance* <*has*> *never failed in perceiving at each moment, if not the final ends of my way* <. . . >, *at least the most fertile available directions leading straightforwardly towards the essential things* <. . . >.

[P.21; main text and n.] *Among the many new points of view that I have developed in mathematics, there are **twelve**, in retrospect, that I would call "big ideas". <...> Here they are:*

1. *Topological Tensor Products and Nuclear Spaces;*
2. *"Continuous" and "Discrete" duality <...>;*
3. *The Riemann-Roch-Grothendieck Yoga (K-Theory and its relationship to Intersection Theory);*
4. *Schemes;*
5. *Topos;*
6. *Etale and ℓ-adic Cohomology;*
7. *Motives, Motivic Galois Groups <...>;*
8. *Crystals, Crystalline Cohomology, yoga of the de Rham and Hodge coefficients;*
9. *Topological Algebra: ∞-stacks, derivations, cohomological formalism of topos, as an inspiration for a new homotopical algebra;*
10. *Moderate topology;*
11. *The yoga of Anabelian Algebraic Geometry and Galois-Teichmüller Theory;*
12. *"Schematic" or "Arithmetic" point of view for regular polyhedra and all regular configurations.*

[P.23] *These twelve major themes of my work are in no way isolated from one another. They are, in my opinion, part of a* ***unity*** *of spirit and purpose, present, like a common and persistent background note, throughout my "written" and "unwritten" work. <...> <They> are all, as if by a secret predestination, contributing to the same symphony <...>, they embody so many different "points of view", all contributing to the same broad* ***vision****.*

[P.21; n.] *Among these themes, the* ***broader*** *one by its* ***scope*** *seems to me to be that of* ***topos****, which provides the idea of a synthesis of algebraic geometry, topology and arithmetic. The broadest one for the* ***extent of the developments*** *to which it has given rise as of now <January 1986> is the theme of schemes. <...> At the opposite extreme, the first and the last of the twelve themes appear to me as being of more modest significance than the others. <...> The deepest <...> are that of* ***motives and that closely related one of the anabelian algebraic geometry and of the Galois-Teichmüller yoga****. Concerning the* ***powerfulness of the tools*** *<...> and of common use in various "advanced sectors" of research during the last two decades <'60-'70>, the ones of "****scheme****" and "****etale and ℓ-adic Cohomology****" appear to me as the worthiest.*

This should make the term 'G's Foundational Program' fully justified,

- and in line with the previous description of what G was taking as Foundation of Mathematics.

Numbers, Magnitudes and Forms

[P.25-26] *Traditionally, we distinguish three types of "qualities" or "aspects" of things in the Universe that are the object of mathematical reflection: these are the* ***number****, the* ***magnitude****, and the* ***form****. We can also term them 'the "****arithmetic****", the "****metric****"', and 'the "****geometric****" aspect' of things. In most situations studied in mathematics, these three aspects are present simultaneously and in close interaction. However, more often than not, there is a marked predominance of one of the three.*

[P.28] *We might say that the "number" is apt of grasping the structure of "discontinuous", or "****discrete****" aggregates* <. . . >. *"Magnitude" is instead the quality par excellence susceptible of "****continuous variation****"; by this, it is apt to grasp continuous structures and phenomena* <. . . >.

It were not for the distinction between magnitude and form (or analysis and geometry), this description would well adapt to ancient and early-modern mathematics, provided the relevant numbers are for G,

> [P.25; n.] *the "numbers" termed 'natural integers'* $<\ldots>$ *or at most the numbers (such as fractional ones) which are expressed with the help of these by operations of elementary nature* $<\ldots><$*, namely*$>$ *numbers* $<$*that*$>$ *do not lend themselves, like the "real numbers" do, to measure a quantity susceptible of continuous variation* $<\ldots>$.

Ancient Greek mathematics was marked by a separation between

- a theory of magnitudes
- and a theory of numbers.

A long-term foundational program in early-modern and enlighten mathematics aimed at an advantageous unification of them.

- This and other subsequent remarks suggest G has been engaged in a similar program, though in a new and innovative context.

By far, the most relevant achievement of the early-modern program was the emergence of a third, unifying, territory: that of analysis, and its forms.

- G seems also to conform with this achievement,
- which makes his program as classic as possible.

But, again, despite his using a very common word in modern mathematics (such as 'structure'), the perspective in which he follows this path is quite original.

Structures

Here what he writes:

> [P.26] <. . . > *if there is one thing in mathematics that (since always, assumably) fascinates me more than any other, it is neither "the number", nor "the magnitude", but always the* ***form****. And among the thousand and one faces that form chooses to reveal itself to us, the one that has fascinated me more than any other and continues to do so is the hidden "****structure****" of mathematical things.*

Notice, indeed, that what G is referring to are not (or, at least are not presented as) theoretical structures—be they algebraic or (putatively) categorical ones—but

- structures of things themselves; immanent structures in the "Universe",
  - structures that we have not to define, but rather to discover, in these very things.

This is as clear as possible in what follows.

> [P.27] *The structure of a thing is not something we can "invent". We can only patiently update it, humbly get to know it, "**discover**" it. If there is inventiveness in this work* $<\ldots>$*, it is by no means to "shape" or "build" "structures". These structures did not wait for us to be, and to be exactly what they are! But it is to express these things as faithfully as we can* $<\ldots>$*. Thus we are led to constantly "**invent" the language** able to express more and more finely the intimate structure of the mathematical thing, and to "build" with the help of this language* $<\ldots>$ *the "theories" which are supposed to give an account of what has been apprehended and seen. There is a continuous, uninterrupted back and forth movement between **apprehension** and **expression** of things* $<\ldots>$*.*

Hence,

- language and theories are invented or constructed;
- structures are discovered and described.

The New Geometry

The task to be accomplished was of carrying out a

> [P.28] *new geometry* $<\ldots>$ *<realizing> the marriage of number and magnitude.*

This marriage also appears to be the final issue (or aim?) of G's unificatory program:

> *[ibidem] This vast unifying vision can be described as a* ***new geometry****. This is the one Kronecker dreamed of in the last century.*

In a footnote, G confesses of only knowing Kronecker dreams by hearsay. It is not clear to me whether he really wanted to refer to the so called *Kronecker's Jugendtraum*, i.e. to Hilbert 12th problem (about the Abelian extension of an algebraic number field), or he was simply using Kronecker's names to decorate his own program with an historical reference.

- The following quotes should in any case make clear the nature of this program.

[P.28] *<...> <while> arithmetic appears <...> as the* ***science of discrete structures****, and analysis as the* ***science of continuous structures*** *<...> we might say that in the more than two thousand years in which it has existed <...> geometry has been "straddling" these two types of structures. <...> For a long time <...> there was no real "****divorce****" between* ***two*** *geometries <...>, one discrete, the other continuous. Rather, there were two different points of view in the investigation of the* ***same*** *geometric figures: one emphasizing "discrete" properties (and in particular, numerical and combinatorial properties), the other emphasizing "continuous" properties (such as position in the surrounding space, or "magnitude" measured in terms of mutual distances of its points, etc.).*

As flawed as it might be by a backwards projection of Gauss's approach on previous geometry, this description is used for shaping a mythological past to which it would be good to go back. Here is, indeed, as G go ahead.

[P.29-31] *It is at the end of the last century that a divorce appeared, with the appearance and the development of what was sometimes called "**abstract** (algebraic) **geometry"**. Roughly speaking, this consisted in introducing, for each prime number* p*, a(n algebraic) geometry "of characteristic* p*", inspired by the (continuous) model of the (algebraic) geometry inherited from the previous centuries, but in a context, however, which appeared as irreducibly "discontinuous", "discrete".* $<\ldots>$ *One can consider that the new geometry is, before anything else, a synthesis between these two worlds, until then adjoining and closely interdependent, but yet separated: the **"arithmetical" world**, in which live the (so-called) "spaces" without principle of continuity, and the **world of continuous magnitude**, where live the "spaces" in the proper sense of the term* $<\ldots>$*.* ***In the new vision, these two formerly separate worlds become one****.* $<$*This is the*$>$ *vision of the "arithmetical geometry".* $<\ldots>$ *The two crucial key ideas in the start-up and development of the new geometry were that of **scheme** and that of **topos**.*

Unification and Recasting

Entering the way these two notions of scheme and topos are used by G to shapes his new "arithmetical geometry" goes, of course, beyond the thematic limits of my patchwork of quotes. All that I can do is

- Appealing to a few quotes to make us feel the unitary afflatus of G's use of these notions.
- Trying to suggest a general methodological background in which this afflatus seems to be embodied.

The background I suggest is provided by

- the notion of (conceptual) recasting.

This is in no way a G's notion. It has been rather suggested to me, in a complete different context, by Ken Manders, and it is my view that it can be used to productively understand a crucial aspect of mathematical activity. I see G's reconstruction of his own foundational program as an authoritative confirmation of this insight.

A quite perspicuous way to introduce this idea is the following quote.

> [P.38-39] *It is this sort of "measurement superstructure", called 'category of sheaves' $<\ldots>$, which will henceforth be considered as "incarnating" what is most essential to space. This is indeed lawful (for the "mathematical common sense"), for it turns out that one can "reconstitute" from scratch a topological space in terms of this associated "'category of sheaves" $<\ldots>$. Nothing else is required $<\ldots>$ to be assured that we can now "forget" the initial space, to retain and use only the associated "category" $<\ldots>$, which will be considered the most adequate incarnation of the "topological" (or "spatial") structure we are trying to express. As so often in mathematics, we have succeeded here (thanks to the crucial idea of "sheaf" $<\ldots>$) in expressing a certain notion (that of "space" in this case) in terms of another (that of "category"). Each time, the discovery of such a* ***translation*** *of a notion $<\ldots>$ in terms of another $<\ldots>$, enriches our understanding of both notions, by the unexpected confluence of the specific intuitions that relate to either one or the other.*

It is what G calls here 'incarnation' or 'translation' and summon by means of the verb 'to express' that I rather suggest to call ' recasting'. Since:

- it is not properly a (faithful) translation, insofar as it involves a crucial shift;
- nor it is an expression, insofar as this shift is not a change of status;
- and it is no more an incarnation, insofar as it is not just a question of adding flesh where there is nothing but spirit.

It is rather a question of transforming a flesh in another flesh by conserving an intellectual content, while regarding it otherwise, within a new conceptual stance, which connect this content to others that did not go with it in the previous stance.

- This is just what makes understanding increase:
    - The disclosure of a new form of presentation going together with a new configuration of contents.

My claim is that

- mathematics often and crucially advances by recasting, and that this is just the purpose of G's foundational program.

The way G goes ahead in his description confirms this reading:

> [P.39] *Thus, a situation of a "topological" nature (incarnated in a given space) is <...> translated into a situation of an "algebraic" nature (incarnated in a "category"); or, if one wishes, the "continuum" incarnated in space, is "translated" or "expressed" through the structure of category, of an "algebraic" nature (and until then perceived as having an essentially "discontinuous" or "discrete" nature).*
>
> *But here, there is more. The <...> notion <...> of space, had appeared to us as a sort of "maximal" notion—a notion so general already, that it is hard to imagine how to find an extension of it that remains "reasonable". On the other hand <...> these "categories" <...> on which we fall, starting from topological spaces, are of a very particular nature. <...> A "new style space"" (or topos), generalizing the traditional topological spaces, will be described simply as a "category" which, without necessarily coming from an ordinary space, nevertheless has all those good properties <...> of such a "category of sheaves".*

Schemes and Topos

Let us came back now, quickly to the role of the notions of schemes and topos. First that of scheme:

> [P.33] *The notion of a scheme is the most natural* $<\ldots>$ *one for encompassing in a single notion the infinite series of notions of (algebraic) "variety"* $<\ldots>$*. Moreover, one and the same "scheme"* $<\ldots>$ *gives rise to a well-determined "(algebraic) variety of characteristic* p*", for each prime number* p*. The collection of these different varieties of the different characteristics can then be visualized as a kind of "(infinite) fan of varieties"* $<\ldots>$*. The "scheme" is this magical fan* $<\ldots>$*. It provides an efficient "principle of passage" to link together "varieties" belonging to geometries that until then had appeared as more or less isolated* $<\ldots>$*. Now, they are included in a common "geometry" and linked by it. One might call it the '**schematic geometry**' , the first draft of that "arithmetical geometry" into which it was to blossom in the following years.*

That of topos, then:

> [P.33,-34] *Each of these geometries remained, however, essentially "discrete" or "discontinuous" in nature, in contrast to the traditional geometry inherited from the past centuries (going back to Euclid).* $<\ldots>$ *What was still missing was clearly some new principle, which would make it possible to link these geometric objects (or "varieties", or "schemes") to the usual (topological) "spaces"* $<\ldots>$.

[P.37-38] *The essential innovative idea was that of (abelian)* ***sheaf*** *on a space, to which Leray associates a sequence of corresponding "cohomology groups" $<\ldots>$. The point of view and the language of sheaves introduced by Leray led us to look at "spaces" and "varieties" of all kinds in a new light. They did not touch, however, the very notion of space, $<\ldots>$ But it turned out that this notion of space is inadequate to account for the most essential "topological invariants" $<\ldots>$. For the expected "marriage", of "number and magnitudes", it was like a too narrow bed, where only one of the future spouses $<\ldots>$ could at best find a place $<\ldots>$ The "new principle" that remained to be found $<\ldots>$ was nothing but this spacious "bed" that was missing to the future spouses $<\ldots>$. This "double bed" appeared$<\ldots>$ with the idea of topos. This idea embraces, in a common topological intuition, both the traditional (topological) spaces, embodying the world of continuous magnitude, and the (so-called) "spaces" (or "varieties") of $<\ldots>$ abstract algebraic geometers, as well as innumerable other types of structures $<\ldots>$.*

Unsettling the Notion of Space

My use of a previous quote aiming at introducing the notion of recasting, might have hidden a much more relevant aspect of this same quote:

- G's claim of having offered a new form of presentation for the notion of space.

This clam is much more explicit in G's comparison of his "contribution to mathematics" and Einstein's one to physics:

> [P.59-60] *both works are accomplished under cover of a* ***mutation of the conception we have of "space"*** $<\ldots>$*; and both take the form of a unifying vision, embracing a vast multitude of phenomena and situations that until then had appeared separate from one another.* $<\ldots>$ *My work has been that of a mathematician* $<\ldots>$ *driven by his very particular genius to constantly enlarge the arsenal of notions at the very basis of his art. This is how I was led* $<\ldots>$ *to upset the most fundamental among all the notions available to the geometer: that of* ***space*** $<\ldots>$*, that is to say our conception of the very "****place****" where geometric beings live.*

Two Final Questions

There are much more notable things in G's *Promenade*, including a more comprehensive account of his achievements, going beyond the notions of scheme and topos. But I cannot but stop by patchwork here.

- But still not my questioning.

Since before concluding, I'd like to ask two questions to a so qualified audience: the former, much more local than the latter.

The first question is this:

- G's main achievements makes use of a category theoretical setting.
  - More than that, his claim of having offered a new form of presentation for the very notion of space depends on the idea that (topological) space is recast by the "category of sheaves"
- What I'm wondering is then this:
  - Is this an essential feature or rather an accidental covering of these achievements?
  - In other terms, would it possible to do, essentially speaking, G's mathematics without category theory?

The second question is not only more global, but also much more provocative.

- G's mathematics is, as it were, a highly conceptual mathematics.
  - It crucially depends on shaping new "notions", "great ideas" and "visions", allowing to constitute a systematic conceptual setting as much general as unified and unifying, and overall mathematically pure, as linear as possible, and free of any local trick or slight.
- What I'm wondering is then this:
  - Is this the sort of mathematics that is really practiced today in the largest part of the mathematical community?
  - Is this, so to say, the style and approach of today's mathematicians?
  - And is this the sort of mathematics that really satisfies the thirst for knowledge of most of them?

Do we not assist, today, to a sort of split, so to say, between a more Eulerian, and less Dedekindian, Hilbertian and Grothendieckian mathematics, almost a counterposition of

- a mathematics made of calculations, local solutions, approximations, experimentations, trials and errors,
- and another one seeking more general theorems and less particular results?

This is far from being a plea for such another (less aristocratic and more plebeian) mathematics, which would be, by the way, possibly unable to develop without the contemporary advances on a most general (Grothendieckian) level.

- It is a genuine question.

And if the answer is positive, I leave you to decide whether this is the case is to complain, or to rejoice.

- In my quality of a philosopher and an historian of mathematics, what I can do is not judging mathematics, but only try to understand its intellectual end epistemic nature.

Thank you for your attention !