

# Investigating Definability in Propositional Logic via Grothendieck Topologies and Sheaves

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**Abstract.** This talk brings together contributions made over the years concerning various topics in propositional logic, including results obtained in cooperation with M. Zawadowski, L. Santocanale, L. Carai. We mostly consider *definability* question like: what is in fact hidden in the seemingly poor language of propositional logic? We mostly concentrate on the case of intuitionistic propositional calculus (IPC). As we will see, definability problems are often related to the task of *solving equations* in appropriate free or extension algebras. Our questions are formulated in *syntactic terms*; despite their purely symbolic nature, investigating them can take benefit from embeddings into *geometric* environments. *Sheaves* over *Grothendieck topologies* supply such environments, to be coupled with appropriate *combinatorial* components (*Ehrenfeucht-Fraïssé Games*).